Doubly Convolutional Neural Networks

SMAI PROJECT

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The Muffin Stuffers

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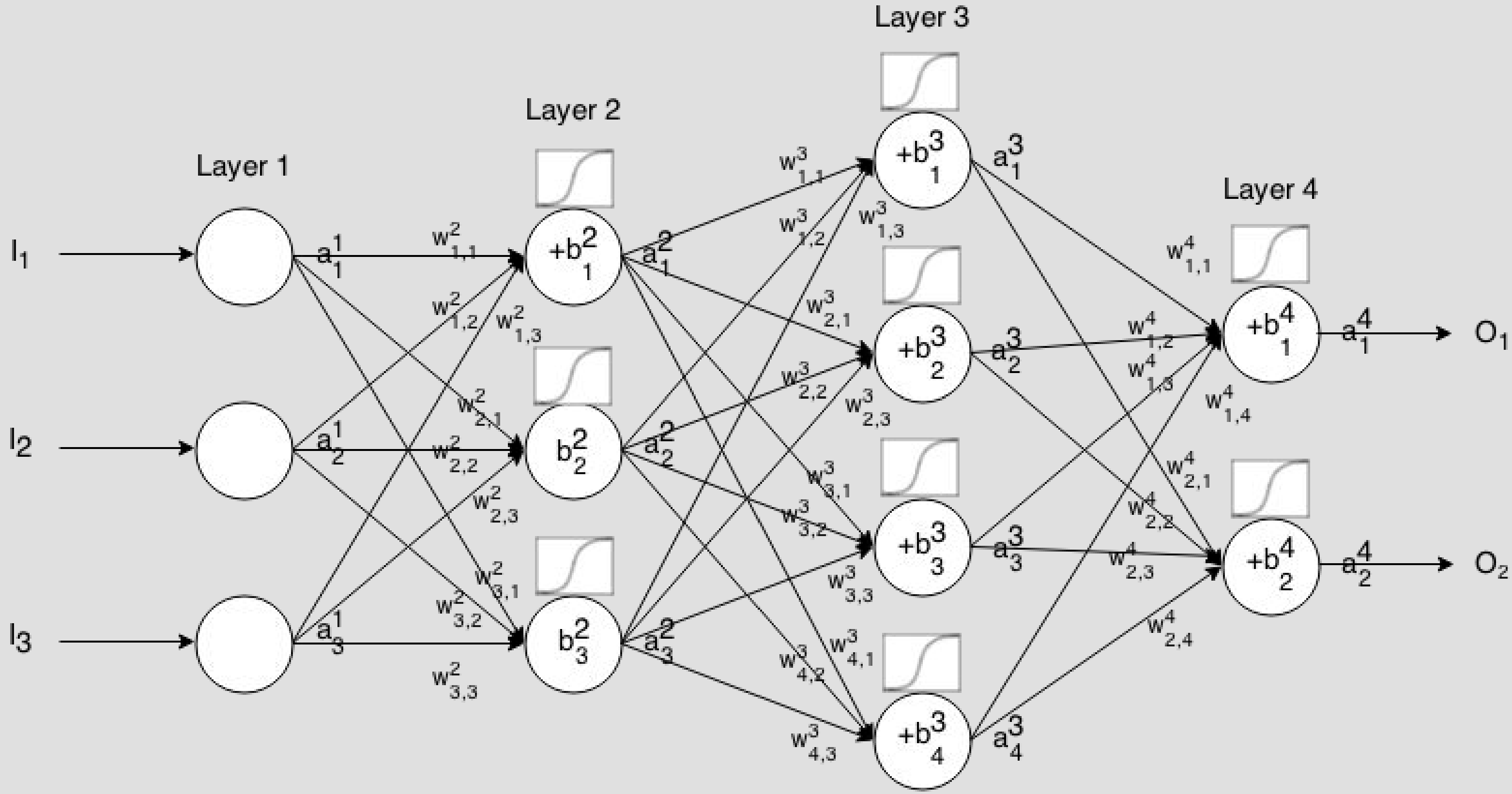
AIM

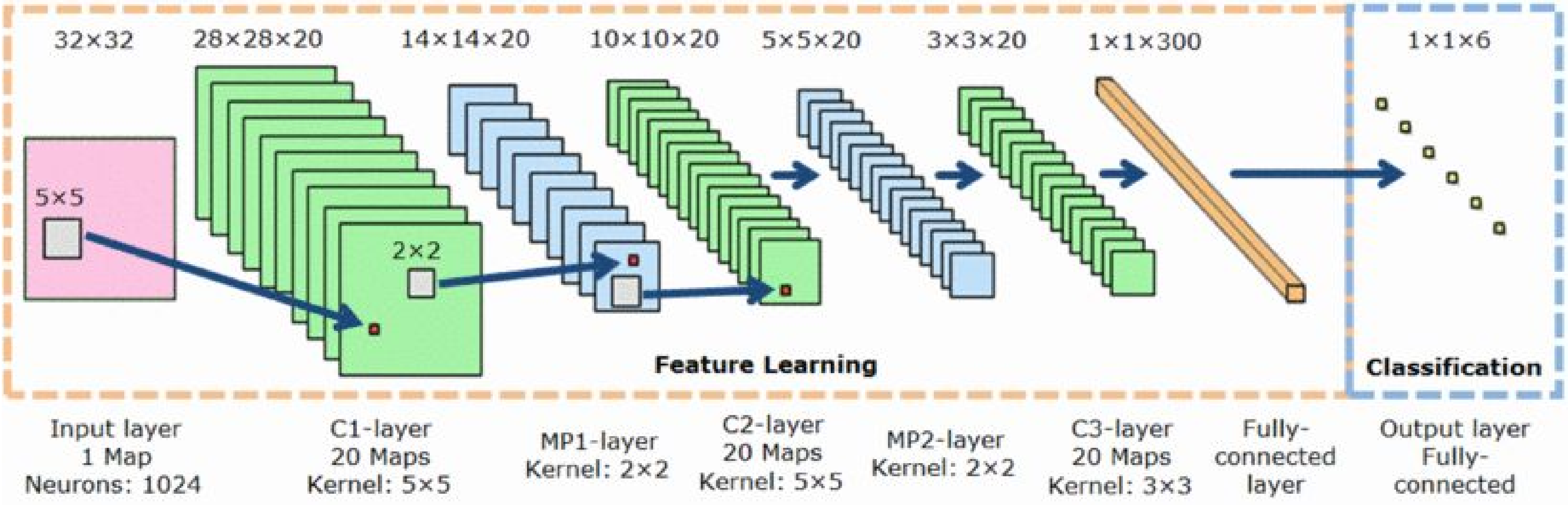
Parameter sharing is the major reason of success of building large models for deep neural networks. This paper introduces the idea of **Doubly Convolutional Neural Networks**, which significantly improves the performance of CNN with the same number of parameters.

Neural Network



Convolutional Neural network

CNNs are extremely parameter efficient due to exploring the translation invariant property of images, which is the key to training very deep models without severe overfitting.



K-Translation Correlation

In well trained CNNs, many of the learned filters are slightly translated versions of each other.

K-translation correlation between two convolutional filters within same layer W_i, W_j is defined as:

$$\rho_k(\mathbf{W}_i, \mathbf{W}_j) = \max_{x,y \in \{-k,\ldots,k\},(x,y)\neq(0,0)} \frac{< \mathbf{W}_i, T(\mathbf{W}_j, x, y) >_f}{\|\mathbf{W}_i\|_2 \|\mathbf{W}_j\|_2}$$

Here, T(.,x,y) denotes the translation of the first operand by (x,y) along its spatial dimensions.

K-translation correlation between a pair of filters indicates the maximum correlation achieved by translating filters up to k steps along any spatial dimension.

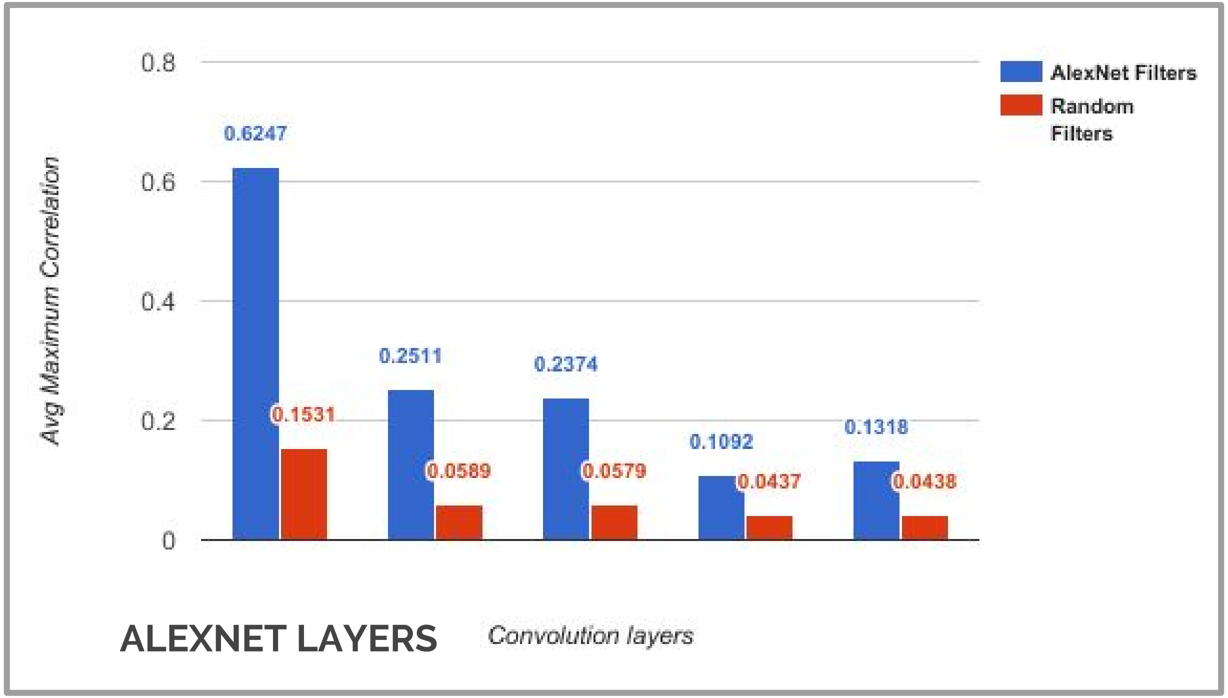
For deeper models, averaged maximum k-translation correlation of a layer **W** is:

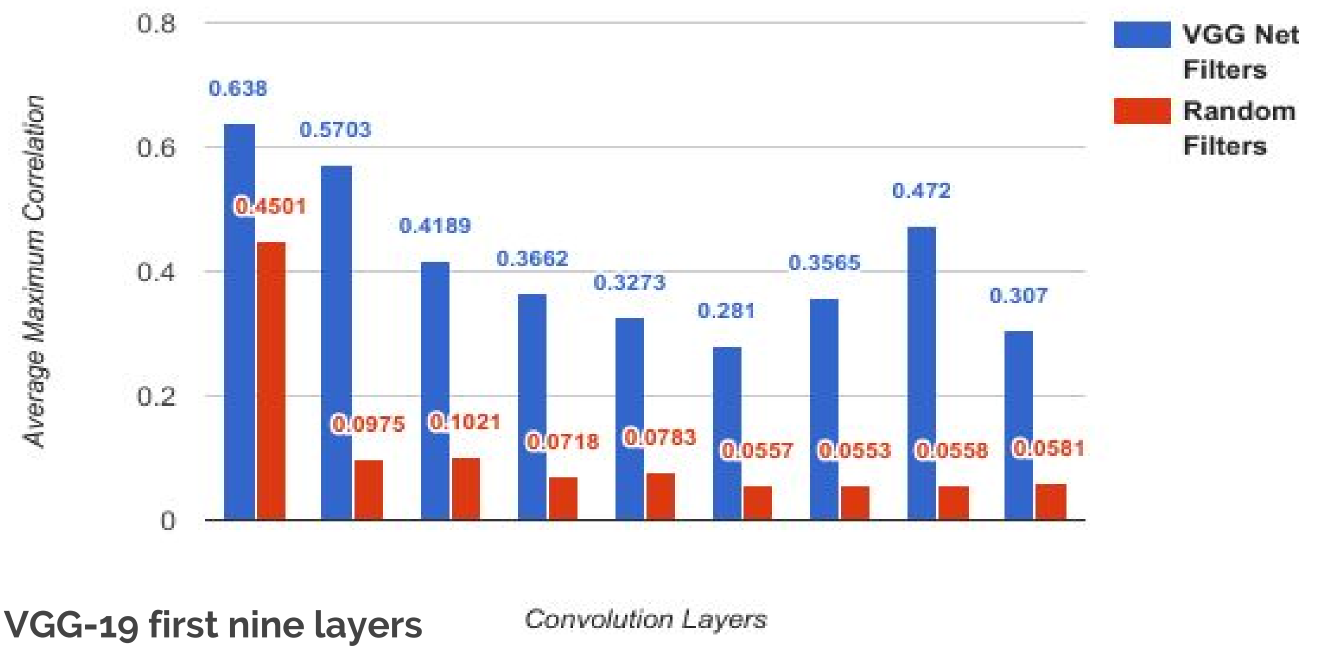
$$\bar{\rho}_k(\mathbf{W}) = \frac{1}{N} \sum_{i=1}^{N} \max_{j=1, j\neq i}^{N} \rho_k(\mathbf{W}_i, \mathbf{W}_j)$$

N is the number of filters

Correlation Results

The averaged maximum 1-translational correlation of each layer for AlexNet and VGG Net are as follows. As a comparison, a filter bank with same shape filled with random gaussian samples has been generated.





VGG-19 first nine layers

Idea of DCNN

Group filters which are translated versions of each other.

DCNN allocates a set of meta filters

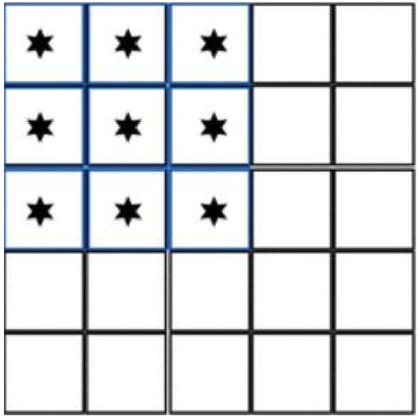
Convolve meta filters with identity kernel

Effective filters extracted

Convolution

$$\mathcal{I}^{\ell+1}_{k,i,j} = \sum_{c'\in[1,c],i'\in[1,z],j'\in[1,z]} \mathbf{W}^{\ell}_{k,c',i',j'} \mathcal{I}^{\ell}_{c',i+i'-1,j+j'-1}$$

$$k \in [1, c^{\ell+1}], i \in [1, w^{\ell+1}], j \in [1, h^{\ell+1}].$$



Input image: $\mathcal{I}^{\ell} \in R^{c^{\ell}\times w^{\ell}\times h^{\ell}}$

Set of c_{l+1} filters : $\mathbf{W}^{\ell} \in R^{c^{\ell+1}\times c^{\ell}\times z\times z}$ ***each filter of shape: c^l x z x z***

Output image: $\mathcal{I}^{\ell+1} \in R^{c^{\ell+1}\times w^{\ell+1}\times h^{\ell+1}}$

Double Convolution

$$\mathcal{O}_{i,j,k}^{\ell+1} = \mathbf{W}_k^{\ell} * \mathcal{I}_{:,i:(i+z-1),j:(j+z-1)}^{\ell},$$

$$\mathcal{I}_{(nk+1):n(k+1),i,j}^{\ell+1} = pool_s(\mathcal{O}_{i,j,k}^{\ell+1}), n = (\frac{z' - z + 1}{s})^2$$

$$k \in [1, c^{\ell+1}], i \in [1, w^{\ell+1}], j \in [1, h^{\ell+1}].$$

Input image: $\mathcal{I}^{\ell} \in R^{c^{\ell} \times w^{\ell} \times h^{\ell}}$

Output image: $\mathcal{I}^{\ell+1} \in R^{nc^{\ell+1} \times w^{\ell+1} \times h^{\ell+1}}$

Set of c^{l+1} meta filters: $\mathbf{W}^{\ell} \in R^{c^{\ell+1} \times c^{\ell} \times z' \times z'}$ *with filter size z'xz', z'>z*

Spatial pooling function with pooling size *s x s*

Working of DCNN

Set of c^{l+1} meta filters size (z' x z')

Image patches size (z x z) convolved with each meta filter

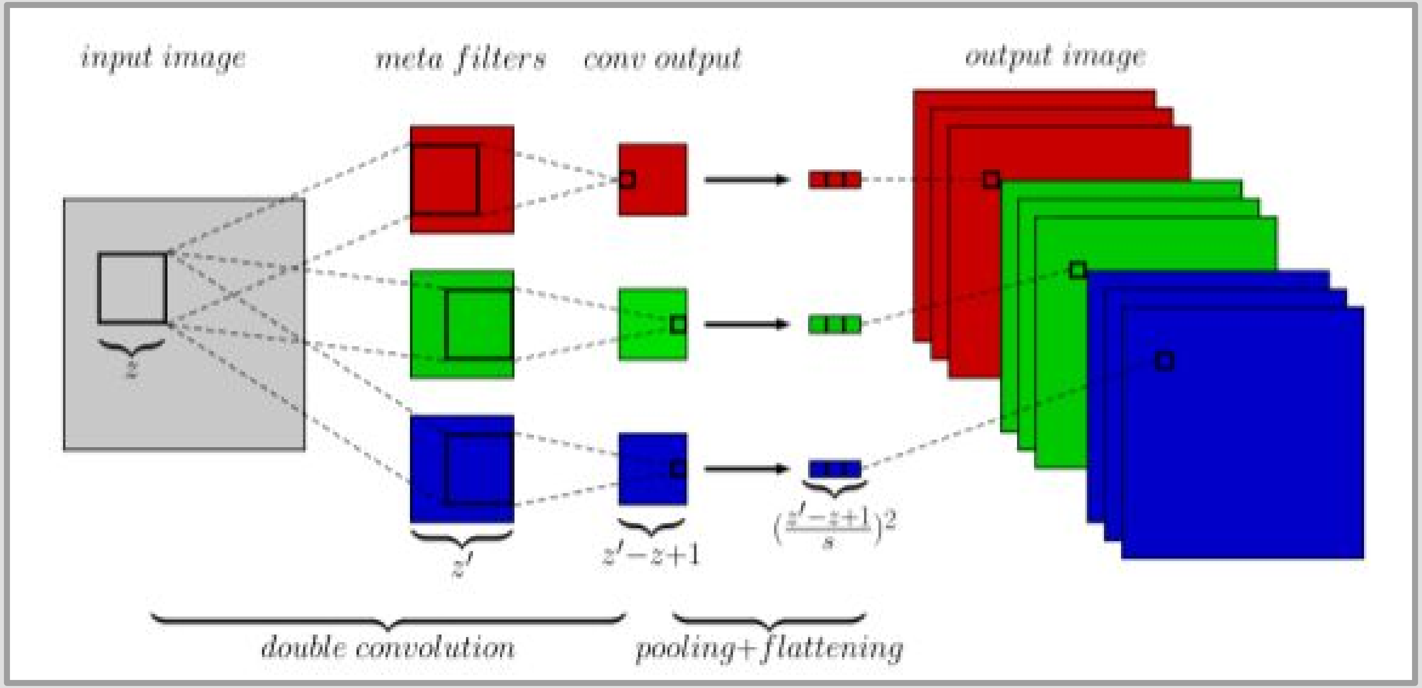
Output size (z'-z+1) x (z'-z+1)

Spatial pooling with size (s x s)

Output flattened to column vector

Feature map with nc^{l+1} channels

Double Convolution: 2 step convolution



STEP1: An image patch is convolved with a metafilter.

STEP2: Meta filters slide across to get different patches, i.e. convolved with the image.

ALGORITHM

Algorithm 1 Implementation of double convolution with convolution.

Input : Input image $I^l \epsilon R^{c^l * w^l * h^l}$, meta filters $W^l \epsilon R^{c^{l+1} * c^l * z' * z'}$, effective filter size $z * z$, pooling size $s * s$.

Output: Output image $I^{l+1} \epsilon R^{nc^{l+1} * w^{l+1} * h^{l+1}}$, with $n = \frac{(z'-z+1)^2}{s^2}$.

function Double Convolution

1. $I^l \leftarrow \ IdentityMatrix(c^l z^2)$;
2. Reorganize I^l to shape $c^l z^2 * c^l * z * z$;
3. $\tilde{W}^l \leftarrow W^l * I^l$; /* output shape: $c^{l+1} * c^l z^2 * (z' - z + 1) * (z' - z + 1)$ */
4. Reorganize $\tilde{W}^l$ to shape $c^{l+1}(z' - z + 1)^2 * c^l * z * z$;
5. $O^{l+1} \leftarrow I^l * \tilde{W}^l$; /* output shape: $c^{l+1}(z' - z + 1)^2 * w^{l+1} * h^{l+1}$ */
6. Reorganize O^{l+1} to shape $c^{l+1} w^{l+1} h^{l+1} * (z' - z + 1) * (z' - z + 1)$;
7. $I^{l+1} \leftarrow pool_s(O^{l+1})$; /* output shape: $c^{l+1} w^{l+1} h^{l+1} * \frac{z'-z+1}{s} * \frac{z'-z+1}{s}$ */
8. Reorganize I^{l+1} to shape $c^{l+1}(\frac{z'-z+1}{s})^2 * w^{l+1} * h^{l+1}$;

end function

1	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	1	0	0	0
0	0	0	0	0	1	0	0
0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	1

Identity (8x8)

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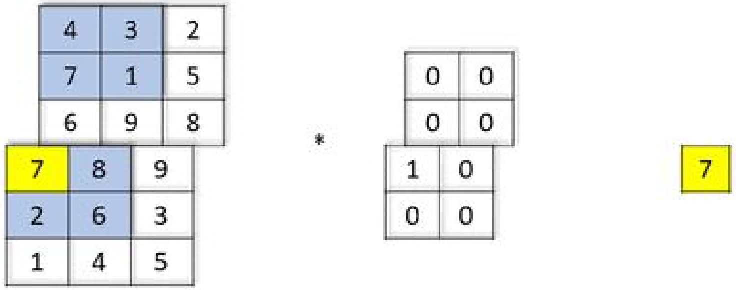
end function

Applying All 8 Masks for fix position



Meta Filter (2x3x3)
Filter (2x2x2) (Blue)

Rearranged 1st Row

Algorithm 1 Implementation of double convolution with convolution.

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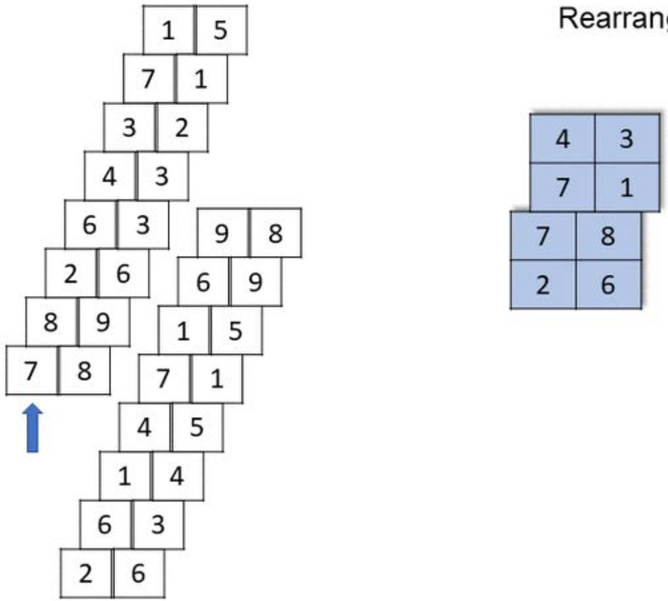
end function

Rearranging



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Implementation & Results

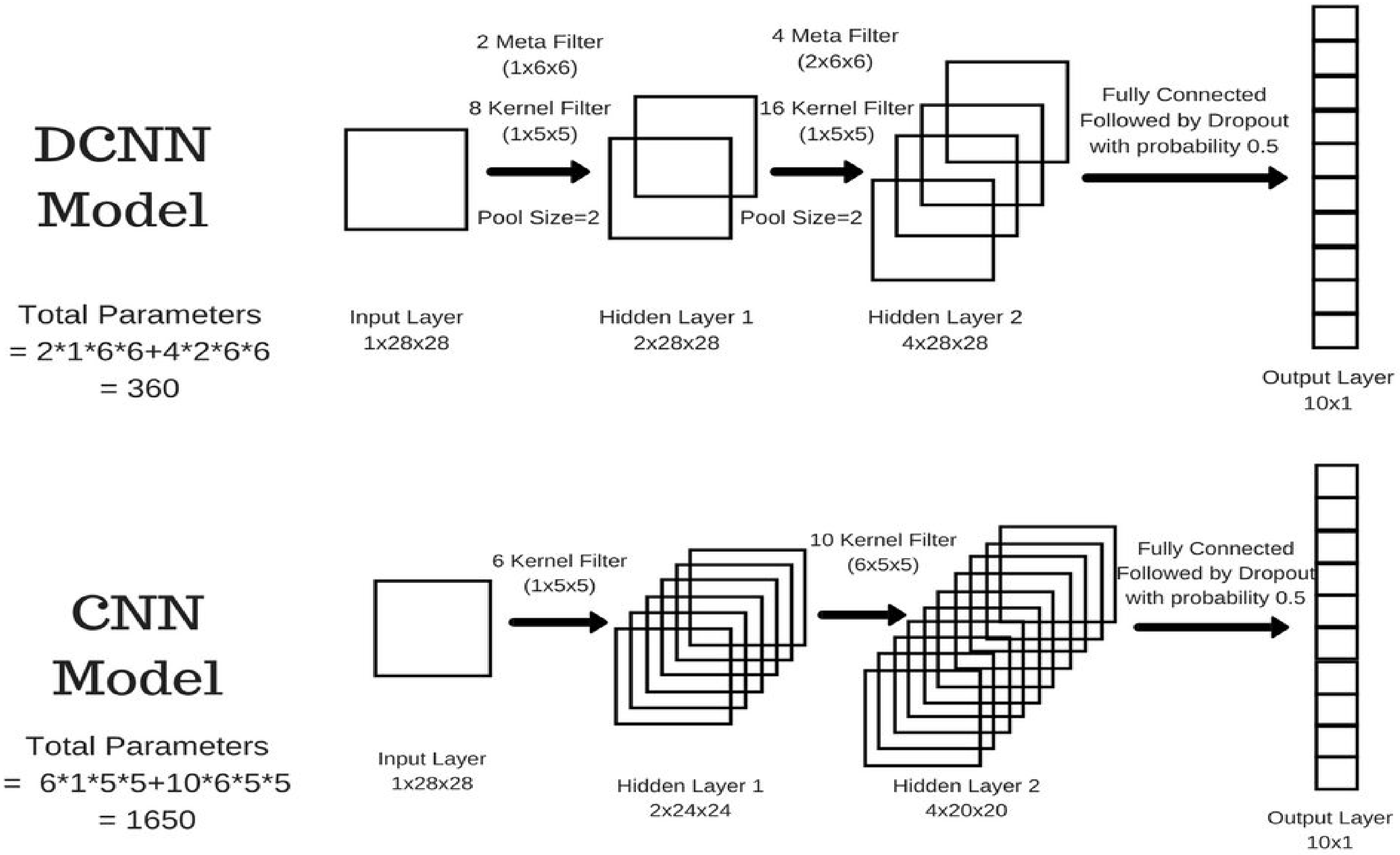
MNIST DATASET

Input: 1x28x28 (GrayScale Image)

Class: 10 (0,1,2, ... , 9)

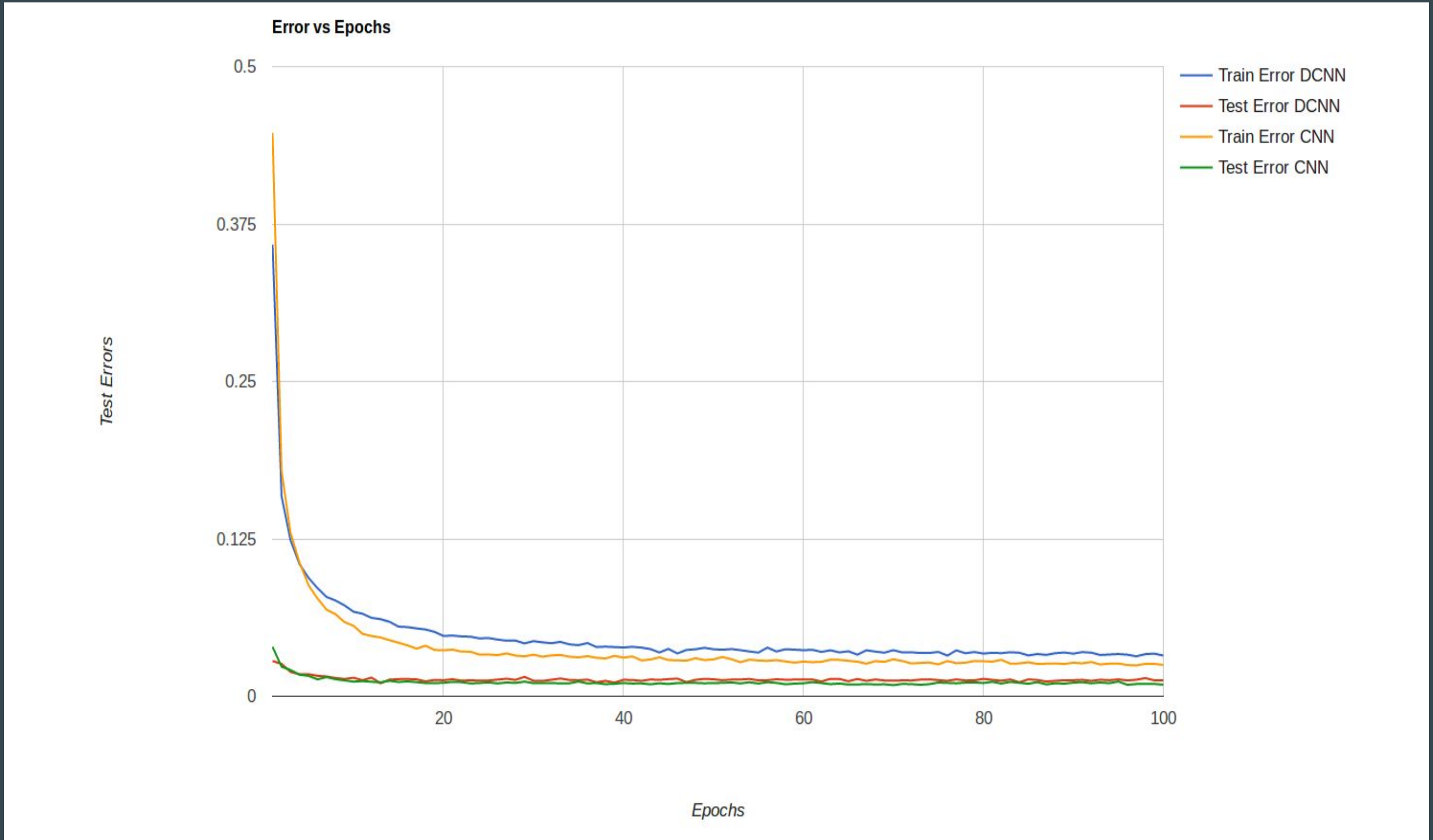
Train Samples: 60,000

Test Samples: 10,000



DCNN Model
Total Parameters
= 2*1*6*6+4*2*6*6
= 360
2 Meta Filter (1x6x6)
8 Kernel Filter (1x5x5)
Pool Size=2
4 Meta Filter (2x6x6)
16 Kernel Filter (1x5x5)
Pool Size=2
Fully Connected Followed by Dropout with probability 0.5
Input Layer 1x28x28
Hidden Layer 1 2x28x28
Hidden Layer 2 4x28x28
Output Layer 10x1
CNN Model
Total Parameters
= 6*1*5*5+10*6*5*5
= 1650
6 Kernel Filter (1x5x5)
10 Kernel Filter (6x5x5)
Fully Connected Followed by Dropout with probability 0.5
Input Layer 1x28x28
Hidden Layer 1 2x24x24
Hidden Layer 2 4x20x20
Output Layer 10x1

Batch Size: 200 Epochs: 100 Dropout: Yes



Minimum Error Values:

DCNN Train: 0.032 at 97

DCNN Test: 0.01 at 13

CNN Train: 0.025 at 97

CNN Test: 0.009 at 70

DCNN vs CNN

Epochs	Pool	Batch Size	Dropout	Test Error DCNN	Test Error CNN
10	2	200	No	0.0137	0.019
9	1	100	No	0.018	0.017
10	2	200	Yes	0.0153	0.0171

Conclusion:
Even though DCNN has 360 params compare to CNN which as 1650 params, Test Error is almost comparable.
Forward Pass Run is Faster in DCNN.
Convergence for DCNN is much faster and after that overfitting happens quickly compare to CNN

Variants of DCNN

Standard CNN

$z' = z$

DCNN is generalisation of CNN

Concat DCNN

$s = 1$

Maximally parameter efficient

With the same amount of parameters produces $\frac{(z'-z+1)^2 z^2}{z'^2}$ times more channels for a single layer.

Maxout DCNN

$s = z' - z + 1$

Output image channel size equal to the number of meta filters.

Yields a parameter efficient implementation of maxout network.

What’s Next?

- Instead of translational correlation modeling for Rotational Correlation.
- Mechanism to decide number of meta filters and its size.

References

- Our Github Repo: https://github.com/tanmayc25/SMAI-Project---DCNN
- Doubly Convolutional Neural Networks (NIPS 2016) by Shuangfei Zhai, Yu Cheng, Weining Lu and Zhongfei (Mark) Zhang

  https://papers.nips.cc/paper/6340-doubly-convolutional-neural-networks.pdf

- Getting Started with Lasagne: http://luizgh.github.io/libraries/2015/12/08/getting-started-with-lasagne/
- Lasagne Docs: https://lasagne.readthedocs.io/en/latest/
- Theano Docs: http://deeplearning.net/software/theano/library/index.html